Acta Neurochir Suppl (2008) 102: 229–233

Pilot application of fractal characterisation and its response to change on physiological wave forms

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Abstract
Background Over the last two decades the advances in analysis techniques for physiological time series data have been moving from the classical statistics to a more nonlinear or chaos based approach to looking at patterns in the variability of the time series. From this work it can be shown that physiological time series exhibit complex multi-fractal properties. So by designing a classification based on this nonlinear and chaotic nature you can detect changes and alterations in the underlying physiological processes.
Methods Applying a proven relationship between the wavelet modulus maxima representation and the Hölder exponent we could assess the multi fractal nature of the of the signal detection underlying changes in the physiology. Using two distinct techniques one global and the other localised in time, classification of two distinct the time series was carried out firstly via the analysis of the distribution of the Hölder exponents over all scales of the signal and secondly via a moving window application of the mean Hölder function.
Findings The distribution methodology did not return significant results though this is probably more to do with the signal than the technique. The trending approach shows a predictive nature with slope being linked to increased instability in the signal content.
Conclusions Overall this study has shown the applicability of the techniques which definitely warrant further refinement and study.

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Keywords Wavelets · Fractals · Mathematical modelling · Wave form analysis · ICP

Introduction

Over the last two decades analysis techniques are moving from classical stochastic process analysis using basic statistics to more nonlinear systems or chaos theoretical based approaches which are looking at patterns in the variability of the time series.

It has been shown that physiological time series exhibit complex multi-fractal properties [1]. So by designing a classification and analysis based on this nonlinear and chaotic nature of the time series we should be able to detect changes and alterations in the underlying physiological processes. To this end two different analysis scenarios were tested. Firstly looking at the overall wave form and characterising its fractal nature and secondly looking at the trending of a window of its fractal nature with time.

Materials

We studied a number of randomly selected ICP waveforms from the BrainIT dataset (http://www.BrainIT.org) and used the R [4] statistical package for algorithm implementation and analysis.

Methods

The main idea used in the initial analysis and fractal characterisation of the wave form leverages the relationship between the mathematical properties of a wavelet transform and a signals' localised fractal nature.

Wavelets [3] can be thought of as a time–frequency analysis technique analogous to a Fourier transform but without the inherent frequency related problems of the later approach. This is mitigated by the fact the convolution is carried out with a scaling factor and a translational factor applied to the convolving function. Like Fourier transforms they can either be continuous or discrete the former implying that all scales are calculated the later only integer scales. The actual transform is carried out via Eq. 1:

$$\mathrm{Wf}(s,b) = \frac{1}{s}\int_{-\infty}^{\infty} f(x)\psi\left(\frac{x-b}{s}\right)\mathrm{d}x. \quad (1)$$

This is a scalable convolution with the ψ function being called a mother wavelet function and the scaled "s" and translated "b" function is known as a daughter function. In all of the later analytic tests a Mexican hat wavelet mother wavelet (Eq. 2) was used:

$$\psi(p) = \left(1-p^2\right)e^{-p/2}, \quad (2)$$

which is the second derivative of the Gaussian function. Figure 1 below is an example analysis using the above mother function on an ICP wave form.

Mandelbrot [2] defines a fractal as self similar signals repeating at different scales within the same signal. He also defines the Hurst exponent as a way of characterisation of the scaling properties of the signal that then can be thought of as an overview of the whole signal. However as we are interested in the signal on a more localised level, there is a related value known as the Hölder exponent [5]. This essentially characterises the singularities of a signal at a single scale level. Where a singularity is defined to be a discontinuity in a signal where the differential of the signal is not continuous at that point. It can be shown that the Taylor expansion [6] of the wavelet transform that Eq. 3 holds over the set of all singularities of the time series:

$$\mathrm{Wf}(s,x_0) \cong |s|^{h(x_0)} \quad (3)$$

where x_0 is the set of all singularities, $h(x_0)$ is defined to be the Hölder exponent of the singularity at x_0 however calculation of "h" by this method is not efficient but by using an optimised partitioning function and counting argument [6] the "local" Hölder exponent can be calculated.

Firstly the wavelet modulus maxima transform (WMMT) is performed and then the ridges are extracted from this view [5]. This ridge representation of the signal is

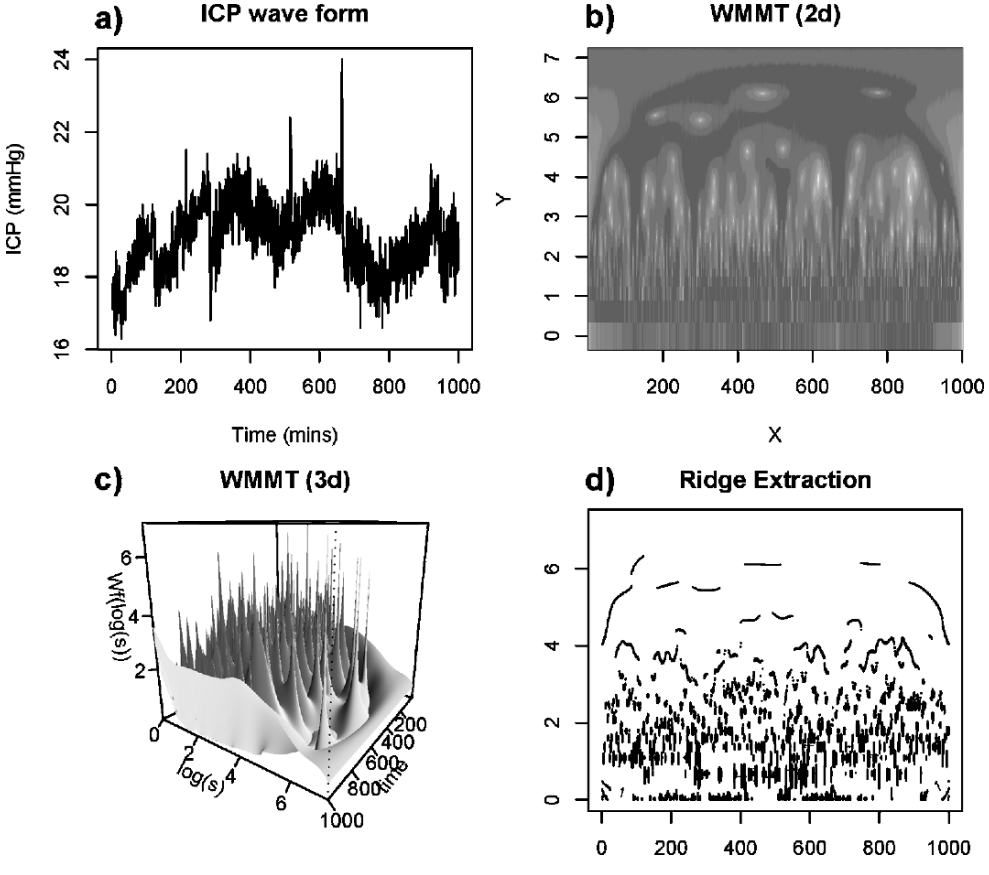

Fig. 1 **a** Original ICP wave form, **b** a 2d wavelet modulus maxima transformation of **a**, **c** a 3d representation of **b**, **d** ridge extraction from **b**

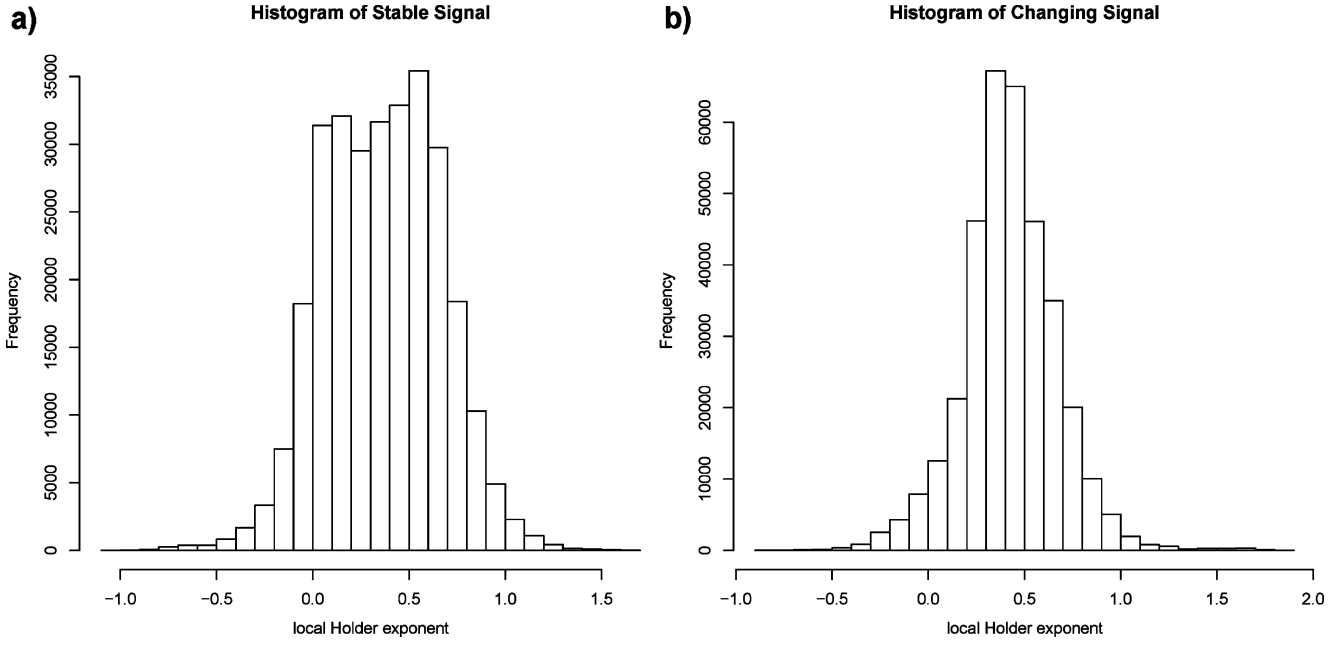

Fig. 2 Comparison of stable (**a**) and changing (**b**) local Hölder exponent distributions over all scales

all that is required to calculate the local Hölder exponent; this should make the analysis more efficient as no redundant information needs to be processed. This form is in fact a representation of all the singularities in the signal. (Fig. 1b,d). Then by using a partitioning function 4 over the set of all singularities $\Omega(s)$ at a given scale "s":

$$Z(s,q) = \sum_{\Omega(s)} (\mathrm{Wf}(\omega_i(s)))^q \tag{4}$$

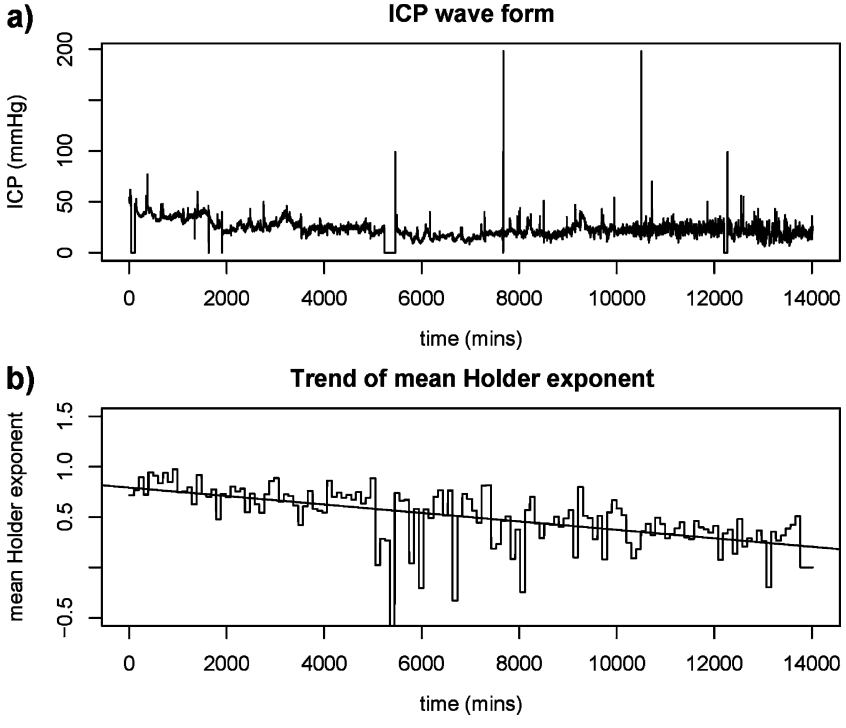

Fig. 3 ICP signal (**a**) and corresponding mean Hölder exponent trend (**b**)

we can define the mean wavelet transform value of all singularities at that scale s to be (Eq. 5):

$$M(s) = \sqrt{\frac{Z(s,2)}{Z(s,0)}}. \tag{5}$$

Finally, the mean Hölder ($\overline{h}$) is then calculated by solving for the slope of the straight line function 6:

$$\log(M(s)) = \overline{h}\log(s) + C. \tag{6}$$

It can then be shown [6] that the local Hölder exponent at scale s is calculated by Eq. 7:

$$\hat{h}_{s_{\mathrm{lo}}}^{s_{\mathrm{SL}}} \cong \frac{\log(\mathrm{Wf}(s_{\mathrm{lo}})) - \log(M(s))}{\log(s_{\mathrm{lo}}) - \log(s_{\mathrm{SL}})} \tag{7}$$

where s_{lo} is the minimum scale used and s_{SL} is the sample length.

This paradigm can be understood in terms that the signal is caused by the physiological system and this signal contains fractal content or information which is directly related to the underlying physiology. This implies that changes to physiology will directly alter the signal and hence the information contained within. By using the above analysis techniques we gain an overview of this fractal information. To test these ideas, two general applications of the above method are presented; Firstly a global signal overview by looking at local Hölder distributions and secondly a more localised view of the data looking at the trending of the mean Hölder exponent.

This first methodology can be thought of as an overview of the signals local Hölder exponent as a global look at the nature of the singularities of the time series and it should present a snap shot of the underlying physiology across the time range something akin to a fingerprint of the signal. ICP wave form data from a random patient was sampled from the BrainIT database and was split into a number of equal lengths. A "stable" segment where the signal remains relatively steady and the amplitude of the number of singularities remain low and a "changing" section, where the signal becomes more unstable and the amount and amplitude of the signal singularities increases. The local Hölder exponent calculated for each of these sections and the distribution of the local Hölder was then plotted and analysed over all scales (Fig. 2).

The second analysis technique to be applied looks at the change or trend in the mean Hölder calculation over the full time course of a signal. It could be thought of as a moving average approach to looking at the Hölder exponents of a signal. This should represent the time course of changes in physiology of the patient.

Again six ICP wave form signals were randomly sampled from the BrainIT dataset and these were then cleaned and a moving window approach was created to allow the repeated application of the mean Hölder function along the time course of the original signal. The window size used for this analysis was 200 min with a 100-min overlap. The choice of this window size was arbitrarily defined by the signal length from previous testing. Once this approach was applied to the signals a linear regression was applied to give an over all trend for the calculated mean Hölder exponents (Fig. 3).

Results

For the first analysis looking at the distributions, as can be seen from (Fig. 2) there is not much if any separation in the distributions and this is representative of all samples tested. In the analysis on trending of the mean Hölder it can be seen to be inversely proportional to signal fractal information content. However quantification of this is difficult as we would need an independent measure of the stability to statistically compare it with the trend gradient. That said over all the samples tested the gradient does show a predictive ability (Fig. 3).

Discussion

In the first analysis the lack of separation is not totally unexpected as the differences in the ICP signals tested between stable and changing sections are not greatly different mathematically, minimally deterministic and based on more random processes and so less likely to have much fractal "information" content. If this is the case then this technique should be then aimed at more deterministic signals such as looking at B wave activity in the ICP trend for example.

In the second analysis the predictive nature of this technique is a promising start though it will require further study to firstly statistically prove and secondly to find the optimal size for the moving window and still provide enough signal to accurately represent it with the mean Hölder exponent.

Overall this pilot study has shown the applicability of the techniques and as such it has only scratched the surface of how these approaches could be applied and of the implications of the links between the physiological systems and the fractal information content of the signals they produce.

Conflict of interest statement We declare that we have no conflict of interest.

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